# 7 Study Guide for Final Exam

Chapter 1 – Understanding Expressions

**Writing Expressions**: If Tara had 3 more than twice as many CDs as Sam, show an expression that illustrates how many CDs Tara has?
Answer: 2x+3

**Writing Equations:** If four quarts (q) equate to one gallon (g), write an equation that shows the relationship between quarts and gallons. The correct answer is 4g=q because you must multiply the number of gallons to determine the number of quarts.

**Order of Operations** (GEMDAS): Evaluate 7 + 8 x 5 = Answer: 47

**Exponents**: Rewrite the expression: $5 \bullet g \bullet g \bullet g = 5g^3$
Then evaluate the expression if g=2, Answer: $5(2)^3=40$

**Using Formulas**: $F = \frac{9}{5}C + 32$ describes the relationship between Fahrenheit and Celsius. If the temperature is 20 degrees Celsius, what is the temperature in Fahrenheit?
Answer: 68 degrees.

**Distributive Property**: n(a+b) = na+nb and n(a-b) = na-nb, so 3(x+4)=3x+12
“Expand the Expression” means to rewrite an expression WITHOUT parenthesis.

Going the opposite direction is called **Factoring,** which means to rewrite WITH parenthesis: 3d+21= 3(d+7)

**Combining Like Terms**: to add or subtract, terms must have the EXACT same variable (letter) and the EXACT same exponent. For example, you CANNOT add 4x + 5 or $3x^2 + 2x$, but you CAN add 5x + 4x to get 9x

Chapter 3 - Signed Numbers

**Absolute Value** is a number’s distance from 0 on the number line. For example:
$|-14| = 14$ and $|14| = 14$

**Adding numbers with like signs**: simply add and keep the sign. For example:
-4 + -6 = -10

**Adding numbers with unlike signs**: Either count on a number line, or use the “team” method. For example: -4 + 7 = 3 because the “negative team” scored 4, the “positive team” scored 7 so the “positive team” wins by 3.

**SITSAATO** – Subtraction Is The Same As Adding The Opposite. Convert subtraction problems with signed numbers into addition problems. For example: 5 - (-4) is the same as 5+4, which equals 9. Similarly, (-2) - (-5) is the same as -2 + 5 which equals 3.

**Addition Expression**: an expression with a '+' sign. Example: a football team gains 5 yards one play, then loses 6 yards the next play 5 + (-6)

**Subtraction Expression**: an expression with a '-' sign. Same example: 5 - 6

**Multiplying signed numbers**:
positive x positive = positive
negative x negative = positive
positive x negative = negative
negative x positive = negative

**Dividing signed numbers**: Rules are identical to those for multiplication.

**Inequality signs:**
x > y means 'x' is greater than 'y'
x < y means 'x' is less than 'y'
x ≥ y means 'x' is greater than or equal to 'y'
x ≤ y means 'x' is less than or equal to 'y'

5 < x < 10 means that 'x' is more than 5 AND less than 10

Solving an inequality is very similar to solving an equation with one MAJOR exception: when we MULTIPLY or DIVIDE by a NEGATIVE number, we must FLIP the sign
For example: -4x > 20 Divide both sides by -4 and FLIP the sign to get x < -5

Chapter 5 – Geometry

The sum of any two sides of a triangle must be greater than the third side. A triangle can be made of 3 sides measuring 4cm, 5cm, and 8cm, but NOT of 4cm, 5cm, and 9cm.

**Prisms** have two identical, parallel faces that are always polygons. A prism's other faces are always parallelograms.
A **triangular prism** has triangular faces on its ends.
A **rectangular prism** has rectangular faces on its ends.

A **cylinder** is like a prism, but its two bases are circles.

**Volume** of a rectangular prism = length x width x height.
For triangular prisms, the formula for volume is ½ x base x height x depth.
For a cylinder, the formula for volume is $\pi r^2 \bullet height$

For **surface area**, add up the area of ALL surfaces of a three dimensional figure.
Rectangular prism, there will be **six** surfaces, all parallelograms.

Triangular prism, there will be **five** surfaces, two triangles and three parallelograms.

Surface area of a sphere = $4\pi r^2$

**Net** is a flattened, 2-D representation of a 3-D shape.

Understand what shape you get when taking a **2-D slice of a 3-D figure**. A slice of a sphere will be a circle, while a slice of a rectangular prism will be a rectangle.

**Congruent figures** have identical angles and side lengths.

**Similar figures** have identical shapes, and proportional side lengths.

**Complementary angles** add up to 90 degrees.

**Supplementary angles** add up to 180 degrees.

Chapter 6 – Data and Probability

**Basic probability** is given by: "successful" outcomes / potential outcomes where "successful" outcome is the outcome being considered. For example, the probability of rolling a 3 on a six-sided die is $\frac{1}{6}$

The probability of an event must be between 0% and 100% (or 0 and 1 in decimal form).

**Compound probability** discusses multiple outcomes of an event. To calculate compound probability, either draw a tree diagram to examine each possible outcome, or if each outcome is **independent**, simply multiply the individual probabilities. For example, the probability of rolling a 3 on a six-sided die twice in a row is $\frac{1}{6} \bullet \frac{1}{6} = \frac{1}{36}$

A **fair game** is one in which all players have an equal chance to score the most points after playing many rounds. One player winning on a 'head' and the other winning on a 'tail' of a flipped fair coin constitutes a fair game. A game played with a single die in which one player gets four points for rolling an even number while the other player gets three points for rolling an odd number is unfair. Both players should earn three points.

**Data Measures**

**Mean**: total up all the data points, then divide by the number of data points
**Median**: the middle value when data points are placed in order by value. If two points are in the middle, then take the mean (or the middle) of the two middle points
**Range**: the distance between the smallest and largest numbers in a data set
**Mode**: the data value seen most often in a data set

**Mean Absolute Deviation** is the average distance from a set of data points to the mean of the data sets. Steps to solve are 1) Find mean of the data set, 2) Calculate the distance of each data point from the mean (all distances must be POSITIVE), 3) Find the mean of the distances by adding them up and dividing by the number of distances.

**Counting Principal:** 4 shirts, 2 pair of pants and 3 pair of socks makes 24 different possible outfits (4 x 2 x 3 = 24)

**Representative Sample** is one in which has approximately the same proportion of characteristics as the entire population.

A **simulation** is an imitation of the operation of a real-world process or system. It can be done manually or by using a computer.

## Chapter 7 – Real Numbers and Pythagorean Theorem

Natural Numbers: 1, 2, 3, 4, …
Whole Numbers: 0, 1, 2, 3, …
Integers: …, -3, -2, -1, 0, 1, 2, 3, …
Rational Numbers: All the above plus fractions and decimals that terminate or repeat
Irrational Numbers: Decimals that do NOT terminate NOR repeat ($\pi$ and most square roots fit in this category)

The **square root** of a number is an amount that when squared produces the number.

Thus, $\sqrt{100} = 10$
There is a positive root and a negative root for all positive numbers.
There is no "real" square root of a negative number.
When solving an equation with a square root, we do the opposite which means we square both sides. Thus, $\sqrt{x} = 7$ we square both sides to get x = 49.

**Pythagorean Theorem**: $a^2 + b^2 = c^2$ in which 'a' and 'b' represent the length of the legs of a right triangle and 'c' represents the length of the hypotenuse. This ONLY applies to right triangles (those with a 90 degree angle).

## Chapter 8 – Linear Relationships

**Linear Relationships** have straight lines when shown on a graph. They are characterized by constant increases in one variable leading to constant increases in another variable.

**Proportional Relationships** describe relationships in which the doubling of one variable leads to a doubling in another variable. When graphed, these relationships pass through the origin. In an equation, these relationships do not have a + or – sign after the slope and 'x' variable, as in y=3x

**Slope** describes the steepness of a line. The larger a positive slope, the steeper the line. A line with a negative slope shows a relationship in which one variable decreases in value while the other variable increases in value. Relationships with negative slopes go "down the hill" when graphed.

A horizontal line has a slope of ‘0’.
A vertical line has an undefined slope.

**Speed** is the same as the absolute value of slope when graphing distance against time.

The **y-intercept** in the point where a line crosses the y-axis (the vertical axis). It is described as (0, x) where ‘x’ is the y-coordinate where the line crosses the y-axis.

**y=mx+b** is the slope-intercept form of a linear relationship in which ‘m’ is the slope, ‘b’ is the y-coordinate of the y-intercept. For example, in the equation y = -3x – 4, the slope is -3 and the y-intercept is (0,-4). ‘x’ and ‘y’ values represent possible solutions to the equation.

## Chapter 9 – Solving Equations

The **Balance method** of solving equations requires that the same thing be done to both sides of an equation to keep it in balance. For example: in the equation $2x+12=58$ we would subtract 12 from both sides, then divide both sides by 2. **The goal is to get ‘x’ all alone on one side.** The order of removing numbers to get ‘x’ along is determined by SADMEG, the opposite of GEMDAS.

Simplifying equations using the distributive property: $3(n+1)=15$ simplifies to $3n+3=15$ because the 3 is “distributed” across EVERYTHING inside the parenthesis.

Subtracting with parenthesis: be careful to “distribute” the negative sign to EVERYTHING inside the parenthesis. For example: $12k-3(7-2k)=12k-21+6k$ Note how the -3 multiplied by the 7 becomes -21 and the -3 multiplied by the -2k becomes +6k.

If a **variable is on BOTH sides** of an equation, SUBTRACT to eliminate the variable from one side. For example: 4x+6=8x-2 our first step is to subtract 4x from both sides to get 6=8x-2.

Solving Inequalities is very similar to solving equations as we also use the balance method. The main EXCEPTION is that when multiplying or dividing by a NEGATIVE number, we must FLIP the inequality symbol so that if faces the OPPOSITE direction. Example: -4x > 20 when we divide both sides by -4 we flip the sign to get x < -5

When graphing inequalities on a number line, we circle the number if the sign is < or > to indicate that the number circled is NOT part of the solution set. We color in the circle if the sign is ≤ or ≥ to indicate the number circled IS part of the solution set.

## Chapter 10 – Ratio and Proportion

A **ratio** shows the relationship between two quantities. The ratio of cows to legs is 4:1, which can also be written as “4 to 1” or as $\frac{4}{1}$. Equivalent ratios have the same value, such as 2:3 and 4:6 or 4 to 1 and 8 to 2.

A **proportion** is two ratios set equal to each other as in $\frac{2}{3} = \frac{4}{6}$

A **unit rate** compares one quantity to one unit of another quantity. 100 miles per two hours is NOT a unit rate, while 50 miles per hour IS a unit rate. By taking different amounts and converting them to unit prices we can compare them. For example: 13 bagels for $3.50 versus 6 bagels for $1.50 converts to $0.27/bagel and $0.25/bagel.

**Proportions** can be used to solve problems like we did in determining the height of a tree: $\frac{5}{6} = \frac{x}{90}$ where a person five feet tall casts a six foot shadow and a tree casts a 90 foot shadow. How tall is the tree? Use cross multiplication or the balance method to solve the problem: x=75

Percentage problems can be solved using proportions:

5 is 20% of what number: $\frac{20}{100} = \frac{5}{x}$, x=25

5 is what percent of 25: $\frac{x}{100} = \frac{5}{25}$, x=20%

what number is 20% of 25: $\frac{20}{100} = \frac{x}{25}$, x=5

Remember, the percentage goes over the 100 and the number following the word "of" goes on the bottom of the opposite side.

**Percentage increases and decreases** are solved after FIRST determining the numerical change and THEN comparing it to the **ORIGINAL** amount. For example: calculate the percentage change between 50 and 65. FIRST solve for the numerical change of 15 by subtracting 50 from 65. THEN compare 15 to 50: $\frac{15}{50} = \frac{x}{100}$

A **markup** is a price INCREASE. For example a retailer who buys a product for a lower price and sells it for a higher price is said to "markup" the price of the product..

A **discount** is a price DECREASE. For example a store offers "discounts" on products by lowering the price.

**Similar figures** have exactly the same shape, but are usually NOT the same size. They have identical angles and corresponding sides with the same ratio, thus the side lengths are *proportional*. Again, we can use proportions to find a missing side.